QCD Sum Rules: Intercrossed Relations for Σ^0 and Λ Magnetic Moments

A.Özpineci, S.B. Yakovlev, and V.S. Zamiralov

NPI MSU Preprint 2003–21/734

MOSCOW 2003

M.V. Lomonosov Moscow State University

D.V. Skobeltsyn Institute of Nuclear Physics

QCD Sum Rules: Intercrossed Relations for Σ^0 and Λ Magnetic Moments

A.Özpineci [1] , S.B. Yakovlev, and V.S. Zamiralov

NPI MSU Preprint 2003–21/734

MOSCOW 2003

[1]International Centre for Theoretical Physics, Trieste, Italy

УДК 539.173

A.Özpineci, S.B. Yakovlev, and V.S. Zamiralov

D.V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia
e-mail: zamir@depni.npi.msu.su
preprint NPI MSU 2003-21/734

QCD Sum Rules: Intercrossed Relations for Σ^0 and Λ Magnetic Moments

Summary

New relations between QCD Borel sum rules for magnetic moments of Σ^0 and Λ hyperons are constructed. It is shown that starting from the sum rule for the Σ^0 hyperon magnetic moment it is straightforward to obtain the corresponding sum rule for the Λ hyperon magnetic moment *et vice versa*.

Резюме

Получены новые соотношения между борелевскими правилами сумм в КХД для магнитных моментов Σ^0 и Λ гиперонов. Показано, что, отправляясь от правила сумм для магнитного момента Σ^0 гиперона, можно непосредственно получить соответствующее правило сумм для магнитного момента Λ гиперона *et vice versa*.

1 Introduction

Recently a series of papers were dedicated to study hadron properties of the $\Sigma, \Sigma_c, \Sigma_b$ baryons as well as of the $\Lambda, \Lambda_c, \Lambda_b$ ones in the framework of various QCD sum rules [1, 2, 3], [4, 5] (we cite only few examples) following pioneer works of [6] and works [7, 8, 9]. Many interesting results were obtained. But full expressions for mass or magnetic moment sum rules become often too long and tedious to achieve and prove. Moreover the Λ hyperon properties is usually treated apart from the other members of the baryon octet. Other Λ-like states also are treated apart from those of the corresponding Σ-like states. Is it possible to relate results for Λ and Σ hyperons among themselves?

We propose here nonlinear intercrossed relations which relate QCD sum rules for magnetic moments of Σ hyperon with that of Λ one and *vice versa* as well as formulae relating sum rules for magnetic moment of Σ or that of Λ with the corresponding sum rule for the $\Sigma^0\Lambda$ transition magnetic moment. Their origin lies in the relation between isotopic, U- and V-spin quantities and is quasi obvious in the framework of the quark model. These relations seem to be useful while obtaining hadron properties of the Λ-like baryons from those of the Σ-like baryons (*et vice versa*) or checking expressions for them mutually.

We begin with a simple example based on the NRQM and then proceed to the QCD sum rules.

2 Relation between magnetic moments of Σ^0 and Λ in the NRQM

Let us write magnetic moments of hyperons Σ^0 and Λ of the baryon octet in the NRQM:

$$\mu(\Sigma^0(ud, s)) = \frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_s; \quad \mu(\Lambda) = \mu_s. \tag{1}$$

As it is known magnetic moment of any other baryon of the octet but that of the Λ hyperon can be obtained from the expression for the Σ^0. E.g., magnetic moment of the $\Sigma^+(uu, s)$ hyperon is obtained just by putting μ_u instead of μ_d in Eq.(1):

$$\mu(\Sigma^+) = \frac{4}{3}\mu_u - \frac{1}{3}\mu_s.$$

But magnetic moment of the Λ hyperon can be also obtained from that of the Σ^0 one, as well as magnetic moment of the Σ^0 can be obtained from that of the Λ one. For that purpose let us formally perform in Eq.(1) the exchange $d \leftrightarrow s$ to get

$$\mu(\tilde{\Sigma}^0_{d\leftrightarrow s}) = \frac{2}{3}\mu_u + \frac{2}{3}\mu_s - \frac{1}{3}\mu_d; \quad \mu(\tilde{\Lambda}_{u\leftrightarrow s}) = \mu_d \tag{2}$$

and the exchange $u \leftrightarrow s$ to get

$$\mu(\tilde{\Sigma}^0_{u\leftrightarrow s}) = \frac{2}{3}\mu_d + \frac{2}{3}\mu_s - \frac{1}{3}\mu_u; \quad \mu(\tilde{\Lambda}_{u\leftrightarrow s}) = \mu_u. \tag{3}$$

The following relations are valid:

$$\begin{aligned} 2(\mu(\tilde{\Sigma}^0_{d\leftrightarrow s}) + \mu(\tilde{\Sigma}^0_{u\leftrightarrow s})) - \mu(\Sigma^0) &= 3\mu(\Lambda); \\ 2(\mu(\tilde{\Lambda}_{d\leftrightarrow s}) + \mu(\tilde{\Lambda}_{u\leftrightarrow s})) - \mu(\Lambda) &= 3\mu(\Sigma^0). \end{aligned} \tag{4}$$

Also

$$\mu(\hat{\Sigma}^0_{d\leftrightarrow s}) - \mu(\hat{\Sigma}^0_{u\leftrightarrow s}) = \sqrt{3}\mu(\Sigma^0\Lambda);$$
$$\mu(\hat{\Lambda}_{d\leftrightarrow s}) - \mu(\hat{\Lambda}_{u\leftrightarrow s}) = -\sqrt{3}\mu(\Sigma^0\Lambda). \tag{5}$$

The origin of these relations lies in the structure of baryon wave functions in the NRQM with isospin $I = 1, 0$ and $I_3 = 0$:

$$2\sqrt{3}|\Sigma^0(ud, s)>_\uparrow = |2u_\uparrow d_\uparrow s_\downarrow + 2d_\uparrow u_\uparrow s_\downarrow - u_\uparrow s_\uparrow d_\downarrow - s_\uparrow u_\uparrow d_\downarrow - d_\uparrow s_\uparrow u_\downarrow - s_\uparrow d_\uparrow u_\downarrow >,$$

$$2|\Lambda>_\uparrow = |d_\uparrow s_\uparrow u_\downarrow + s_\uparrow d_\uparrow u_\downarrow - u_\uparrow s_\uparrow d_\downarrow - s_\uparrow u_\uparrow d_\downarrow >,$$

where $q_\uparrow$ $(q_\downarrow)$ means wave function of the quark q (here $q = u, d, s$) with the helicity +1/2 (-1/2). With the exchanges $d \leftrightarrow s$ and $u \leftrightarrow s$ one arrives at the corresponding U-spin and V-spin quantities, so $U = 1, 0$ and $U_3 = 0$ baryon wave functions are

$$-2|\hat{\Sigma}^0_{d\leftrightarrow s}(us, d)>= |\Sigma^0(ud, s)> +\sqrt{3}|\Lambda>,$$

$$-2|\hat{\Lambda}_{d\leftrightarrow s}>= -\sqrt{3}|\Sigma^0(ud, s)> +|\Lambda>,$$

while $V = 1, V_3 = 0$ and $V = 0$ baryon wave functions are

$$-2\hat{\Sigma}^0_{u\leftrightarrow s}(ds, u) = |\Sigma^0(ud, s)> -\sqrt{3}|\Lambda>,$$

$$2|\hat{\Lambda}_{u\leftrightarrow s}>= \sqrt{3}|\Sigma^0(ud, s)> +|\Lambda> .$$

It is easy to show that relations given by Eqs.(2,3) immeaditely follow.

3 Relation between QCD correlators for Σ^0 and Λ

Now we demonstrate how similar considerations work for QCD sum rules on the example of QCD Borel sum rules for magnetic moments.

The starting point would be two-point Green's function for hyperons Σ^0 and Λ of the baryon octet:

$$\Pi^{\Sigma^0,\Lambda} = i\int d^4x e^{ipx} < 0|T\{\eta^{\Sigma^0,\Lambda}(x), \eta^{\Sigma^0,\Lambda}(0)\}|0>, \tag{6}$$

where isovector (with $I_3 = 0$) and isocalar field operators could be chosen as [4]

$$\eta^{\Sigma^0} = \frac{1}{2}\epsilon_{abc}[(\mathrm{u^{aT}Cs^b})\gamma_5\mathrm{d^c} - (\mathrm{d^{aT}Cs^b})\gamma_5\mathrm{u^c} - (\mathrm{u^{aT}C\gamma_5 s^b})\mathrm{d^c} + (\mathrm{d^{aT}C\gamma_5 s^b})\mathrm{u^c}],$$
$$\eta^\Lambda = \frac{1}{2\sqrt{3}}\epsilon_{abc}[-2(\mathrm{u^{aT}Cd^b})\gamma_5\mathrm{s^c} + (\mathrm{u^{aT}Cs^b})\gamma_5\mathrm{d^c} + (\mathrm{d^{aT}Cs^b})\gamma_5\mathrm{u^c} +$$
$$2(\mathrm{u^{aT}C\gamma_5 d^b})\mathrm{s^c} - (\mathrm{u^{aT}C\gamma_5 s^b})\mathrm{d^c} - (\mathrm{d^{aT}C\gamma_5 s^b})\mathrm{u^c}], \tag{7}$$

where a, b, c are the color indices and u, d, s are quark wave functions, C is charge conjugation matrix.

We show now that one can operate with Σ hyperon and obtain the results for the Λ hyperon. The reasoning would be valid also for charm and beaty Σ-like and Λ-like baryons.

In order to arrive at the desired relations we write not only isospin quantities but also U-spin and V-spin ones.

Let us introduce U-vector (with $U_3 = 0$) and U-scalar field operators just formally changing $(d \leftrightarrow s)$ in the Eq.(7):

$$\hat{\eta}^{\Sigma^0(d\leftrightarrow s)} = \frac{1}{2}\epsilon_{abc}[(\mathbf{u^{aT}}\mathbf{C}\cdot 1\cdot \mathbf{d^b})\gamma_5\mathbf{s^c} - (\mathbf{s^{aT}}\mathbf{C}\mathbf{d^b})\cdot\gamma_5\cdot\mathbf{u^c} - (1\leftrightarrow\gamma_5)]$$
$$\hat{\eta}^{\Lambda(d\leftrightarrow s)} = \frac{1}{2\sqrt{3}}\epsilon_{abc}[(-2(\mathbf{u^{aT}}\mathbf{C}\cdot 1\cdot \mathbf{s^b})\gamma_5\mathbf{d^c} + (\mathbf{u^{aT}}\mathbf{C}\cdot 1\cdot\mathbf{d^b})\cdot\gamma_5\cdot\mathbf{s^c} + (\mathbf{s^{aT}}\mathbf{C}\cdot 1\cdot\mathbf{d^b})\cdot\gamma_5\cdot\mathbf{u^c}) - (1\leftrightarrow\gamma_5)]. \quad (8)$$

Similarly we introduce V-vector (with $V_3 = 0$) and V-scalar field operators just changing $(u \leftrightarrow s)$ in the Eq.(7):

$$\hat{\eta}^{\Sigma^0(u\leftrightarrow s)} = \frac{1}{2}\epsilon_{abc}[(\mathbf{s^{aT}}\mathbf{C}\cdot 1\cdot \mathbf{u^b})\gamma_5\mathbf{d^c} - (\mathbf{d^{aT}}\mathbf{C}\mathbf{u^b})\cdot\gamma_5\cdot\mathbf{s^c} - (1\leftrightarrow\gamma_5)]$$
$$\hat{\eta}^{\Lambda(u\leftrightarrow s)} = \frac{1}{2\sqrt{3}}\epsilon_{abc}[(-2(\mathbf{u^{aT}}\mathbf{C}\cdot 1\cdot \mathbf{s^b})\gamma_5\mathbf{u^c} + (\mathbf{u^{aT}}\mathbf{C}\cdot 1\cdot\mathbf{u^b})\cdot\gamma_5\cdot\mathbf{s^c} + (\mathbf{d^{aT}}\mathbf{C}\cdot 1\cdot\mathbf{u^b})\cdot\gamma_5\cdot\mathbf{s^c}) - (1\leftrightarrow\gamma_5)]. \quad (9)$$

Field operators of the Eq.(7) and Eq.(8) can be related through

$$-2\hat{\eta}^{\Lambda(d\leftrightarrow s)} = \sqrt{3}\eta^{\Sigma^0} + \eta^{\Lambda},$$
$$2\hat{\eta}^{\Sigma^0(d\leftrightarrow s)} = \eta^{\Sigma^0} - \sqrt{3}\eta^{\Lambda},$$
$$2\hat{\eta}^{\Lambda(u\leftrightarrow s)} = \sqrt{3}\eta^{\Sigma^0}\eta^{\Lambda},$$
$$2\hat{\eta}^{\Sigma^0(u\leftrightarrow s)} = \eta^{\Sigma^0} + \sqrt{3}\eta^{\Lambda}. \quad (10)$$

Upon using Eqs.(7-10) two-point functions of the Eq.(6) for hyperons Σ^0 and Λ of the baryon octet can be related as

$$2[\hat{\Pi}^{\Sigma^0(d\leftrightarrow s)} + \hat{\Pi}^{\Sigma^0(u\leftrightarrow s)}] - \Pi^{\Sigma^0} = 3\Pi^{\Lambda}, \quad (11)$$

$$2[\hat{\Pi}^{\Lambda(d\leftrightarrow s)} + \Pi^{\hat{\Lambda}(u\leftrightarrow s)}] - \Pi^{\Lambda} = 3\Pi^{\Sigma^0}. \quad (12)$$

These are essentially nonlinear relations.

It is seen that starting calculations, e.g., from Σ-like quantities one should arrive at the correspondong quantities for Λ-like baryons *et vice versa.*

Moreover one can obtain QCD Borel sum rule for the $\Sigma^0 - \Lambda$ transition magnetic moment using the relations

$$[\hat{\Pi}^{\Sigma^0(u\leftrightarrow s)} - \hat{\Pi}^{\Sigma^0(d\leftrightarrow s)}] = \sqrt{3}\Pi^{\Lambda\Sigma^0}, \quad (13)$$

$$[\hat{\Pi}^{\Lambda(u\leftrightarrow s)} - \Pi^{\hat{\Lambda}(d\leftrightarrow s)}] = -\sqrt{3}\Pi^{\Lambda\Sigma^0}. \quad (14)$$

4 Intercrossed relations for the QCD magnetic moment sum rules

In order to demonstrate clearly how it works we prefered not to use QCD sum rules elaborated by one of us with coauthors [4], [5] though they perfectly satisfy the relations

(11-14), but instead to repeat calculations of the first of the QCD Borel sum rules for magnetic moments following [7] conserving non-degenerated quantities for u and d quarks.

We would need quantities [6, 7]

$$a_q = -(2\pi)^2 < \bar{q}q >, \quad b =< g_c G^2 >,$$
$$a_q m_{0(q)}^2 = (2\pi)^2 < g_c \bar{q}\sigma \cdot Gq >, \quad q = u, d, s.$$
$$< \bar{q}\sigma_{\mu\nu} q >_F = c_q \chi < \bar{q}q > F_{\mu\nu}, \tag{15}$$

while the susceptibilities κ and ξ are defined through

$$< \bar{q} g_s G_{\mu\nu} q >_F = c_q \kappa < \bar{q}q > F_{\mu\nu},$$

$$\epsilon_{\alpha\beta\mu\nu} < \bar{q} g_s G^{\mu\nu} \gamma_5 q >_F = i c_q \xi < \bar{q}q > F_{\alpha\beta}.$$

Also we would define a factor used to subtract the continuum contribution [6]:

$$E_n(x) = 1 - e^{-x}(1 + x + ... + x^n/n!), \quad x = W_B^2/M^2, \quad B = \Sigma^0, \Lambda,$$

although here we do not use it.

We shall construct sum rule for the magnetic moment of the Σ^0 hyperon in the form

$$SR(\Sigma^0) = \sum_{i=1}^{10} \Sigma^{0(i)} = \beta_{\Sigma^0}^2 (\mu(\Sigma^0) + AM^2) e^{-M_{\Sigma^0}^2/M^2} + e.s.c., \tag{16}$$

and then (re)derive term by term the corresponding quantities for the Λ hyperon upon using Eq.(11) and $\Sigma^0 - \Lambda$ transition upon using Eq.(13). Here β_{Σ^0} is a coupling strength of the Σ^0 current to the ground state of this hyperon, while A is a constant arising from the non-diagonal transitions and $e.s.c.$ means 'excited state contriburions.

We would proceed pass by pass to show also that every group of diagrams yields expressions which can be treated through Eqs.(11,13) in autonome way.

The 1st term $\Sigma^{0(1)}$ comes from the first two diagrams of the Fig.1.

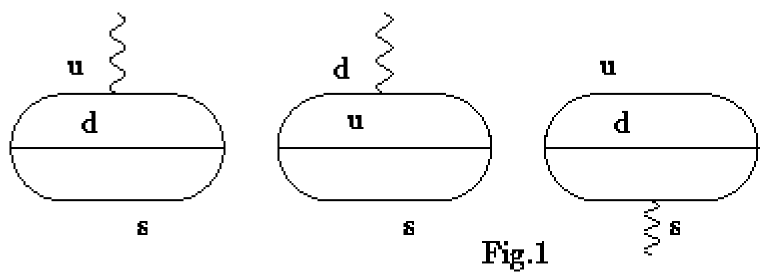

Fig.1

$$2\Sigma^{0(1)} = \frac{M^6}{4L^{4/9}} 2(e_u + e_d),$$

wherefrom the auxiliary quantities $\tilde{\Sigma}_{sd}^{0(1)}$ and $\tilde{\Sigma}_{su}^{0(1)}$ would be

$$2\tilde{\Sigma}_{sd}^{0(1)} = \frac{M^6}{4L^{4/9}} 2(e_u + e_s), \quad 2\tilde{\Sigma}_{su}^{0(1)} = \frac{M^6}{4L^{4/9}} 2(e_d + e_s).$$

Applying Eq.(11) we arrive at the contribution of the Λ hyperon $\Lambda^{(1)}$ given already by all three diagrams of the Fig.1:

$$\Lambda^{(1)} = \frac{M^6}{12L^{4/9}} (e_u + e_d + 4e_s)$$

in agreement with the 1st term of the Eq.(29) in [7]. The difference between these auxiliary quantities just gives up to a factor $\sqrt{3}$ the $\Sigma^0\Lambda$ transition magnetic moment in accordance with the relation (13):

$$\sqrt{3}(\Sigma^0\Lambda)^{(1)} = \tilde{\Sigma}^{0(1)}_{su} - \tilde{\Sigma}^{0(1)}_{sd} = \frac{M^6}{4L^{4/9}}(e_u - e_d),$$

which agrees with the corresponding term in [3].

The 2nd term $\Sigma^{0(2)}$ comes from the diagrams of the Fig.2.

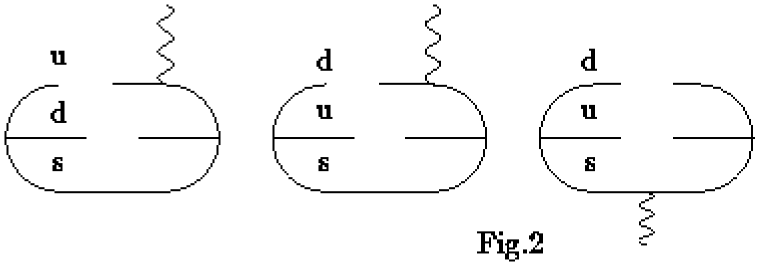

Fig.2

$$2\Sigma^{0(2)} = -\frac{L^{4/9}}{18M^2}a_u a_d 2(e_u + e_d + 3e_s);$$

$$2\tilde{\Sigma}^{0(2)}_{sd} = -\frac{L^{4/9}}{18M^2}a_u a_s 2(e_u + e_s + 3e_d);$$

$$2\tilde{\Sigma}^{0(2)}_{su} = -\frac{L^{4/9}}{18M^2}a_d a_s 2(e_d + e_s + 3e_u).$$

Upon applying Eq.(11) we obtain

$$\Lambda^{(2)} = \frac{1}{3}[2\tilde{\Sigma}^{0(2)}_{sd} + 2\tilde{\Sigma}^{0(2)}_{su} - \Sigma^{0(2)}] =$$

$$-\frac{L^{4/9}}{54M^2}\{2[(e_u a_u + e_d a_d) + 3(e_u a_d + e_d a_u) + e_s(a_u + a_d)] -$$

$$a_u a_d(e_u + e_d + 3e_s)\}.$$

Taking $a_u = a_d = a$, $a_s/a = f + 1$, we get

$$\Lambda^{(2)} = -\frac{L^{4/9}}{108M^2}[2(7e_u + 7e_d + e_s) + 8f(2e_u + 2e_d + e_s)]a^2$$

in agreement with the Eq.(29) in [7].

The difference between these auxiliary quantities yields:

$$\sqrt{3}(\Sigma^0\Lambda)^{(2)} = \frac{L^{4/9}}{18M^2}a_s[(a_u e_u - a_d e_d) + 3(e_d a_u - e_u a_d) + e_s(a_u - a_d)]$$

$$\rightarrow (\quad a_u = a_d = a) - \frac{L^{4/9}}{9M^2}a_s a(e_u - e_d).$$

in agreement with the corresponding term in [3].

The 3nd term $\Sigma^{0(3)}$ comes from the convergent part of the diagrams of the Fig.3-5.

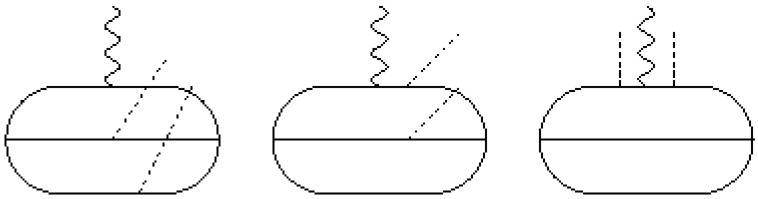

Fig.3 Fig.4 Fig.5

$$2\Sigma^{0(3)} = \frac{M^2 b}{24L^{4/9}}(2c_u + 2c_d + c_s).$$

It is straightforward to obtain from Eq.(11)

$$\Lambda^{(3)} = \frac{M^2 b}{144L^{4/9}}(4c_u + 4c_d + 7c_s)$$

again in agreement with the Eq.(29) in [7]. The difference between these auxiliary quantities gives:

$$\sqrt{3}(\Sigma^0\Lambda)^{(3)} = \frac{M^2 b}{24L^{4/9}}(c_u - c_d).$$

The 4th term comes from the divergent part of the diagrams of the Fig.4, regularized by a cutoff Λ:

$$2\Sigma^{0(4)} = \frac{-M^2 b}{144L^{4/9}}[ln(\frac{M^2}{\Lambda^2}) - 1 - \gamma_{EM}]2(c_u + c_d + 2c_s).$$

It is straightforward to obtain from Eq.(11)

$$\Lambda^{(4)} = \frac{-M^2 b}{192L^{4/9}}[ln(\frac{M^2}{\Lambda^2}) - 1 - \gamma_{EM}](5c_u + 5c_d + 2c_s)$$

again in full agreement with the 4th term of the Eq.(29) in [7].

The difference $\tilde{\Sigma}^{0(4)}_{su} - \tilde{\Sigma}^{0(4)}_{sd}$ yields:

$$\sqrt{3}(\Sigma^0\Lambda)^{(4)} = -\frac{M^2 b}{144L^{4/9}}[ln(\frac{M^2}{\Lambda^2}) - 1 - \gamma_{EM}](c_u - c_d).$$

The 5th term comes from the divergent part of the diagrams of the Fig.5, regularized by a cutoff Λ:

$$2\Sigma^{0(5)} = -\frac{M^2 b}{36L^{4/9}}[ln(\frac{M^2}{\Lambda^2}) - \gamma_{EM} - \frac{M^2}{2\Lambda^2}]2(c_u + c_d).$$

We readily obtain from Eq.(11)

$$\Lambda^{(5)} = -\frac{M^2 b}{108L^{4/9}}[ln(\frac{M^2}{\Lambda^2}) - \gamma_{EM} - \frac{M^2}{2\Lambda^2}](c_u + c_d + 4c_s)$$

in agreement with the result of [7].

The difference $\tilde{\Sigma}^{0(5)}_{su} - \tilde{\Sigma}^{0(5)}_{sd}$ yields:

$$\sqrt{3}(\Sigma^0\Lambda)^{(5)} = -\frac{M^2 b}{36L^{4/9}}[ln(\frac{M^2}{\Lambda^2}) - \gamma_{EM} - \frac{M^2}{2\Lambda^2}](c_u - c_d).$$

The 6th term $\Sigma^{0(6)}$ involves susceptibilities χ, κ, ξ as it is seen from the diagrams of the Fig.6 and is given by

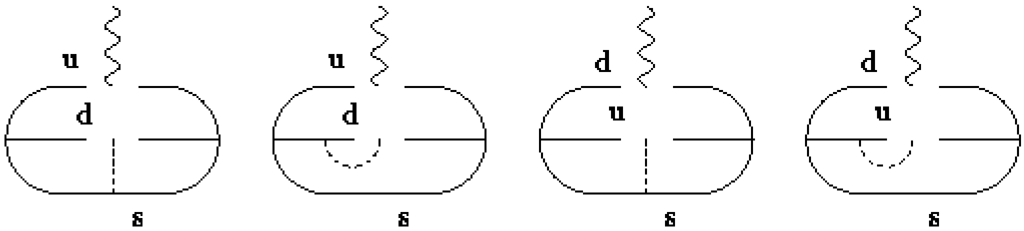

Fig.6

$$2\Sigma^{0(6)} = -\frac{1}{3L^{4/27}}[(M^2 - \frac{m^2_{(d)0}}{8L^{4/9}})c_u(\chi_u a_u)a_d + (M^2 - \frac{m^2_{(u)0}}{8L^{4/9}})c_d(\chi_d a_d)a_u)]+$$
$$\frac{L^{4/9}}{18}[c_u((2\kappa_u - \xi_u)a_u a_d + c_d((2\kappa_d - \xi_d)a_d a_u)],$$

while the auxiliary quantities are:

$$2\hat{\Sigma}^{0(6)}_{sd} = -\frac{1}{3L^{4/27}}[(M^2 - \frac{m^2_{(s)0}}{8L^{4/9}})c_u(\chi_u a_u)a_s + (M^2 - \frac{m^2_{(u)0}}{8L^{4/9}})c_s(\chi_s a_s)a_u]+$$
$$\frac{L^{4/9}}{18}[c_u((2\kappa_u - \xi_u)a_u a_s + c_s((2\kappa_s - \xi_s)a_s a_u)];$$
$$2\hat{\Sigma}^{0(6)}_{su} = -\frac{1}{3L^{4/27}}[(M^2 - \frac{m^2_{(s)0}}{8L^{4/9}})c_d(\chi_d a_d)a_s + (M^2 - \frac{m^2_{(d)0}}{8L^{4/9}})c_s(\chi_s a_s)a_d]+$$
$$\frac{L^{4/9}}{18}[c_d((2\kappa_d - \xi_d)a_d a_s + c_s((2\kappa_s - \xi_s)a_s a_d)].$$

Upon using Eq.(11) we get:

$$\Lambda^{(6)} = -\frac{1}{18L^{4/27}}\{(M^2 - \frac{m^2_{(d)0}}{8L^{4/9}})[2c_s(\chi_s a_s) - c_u(\chi_u a_u)]a_d +$$
$$(M^2 - \frac{m^2_{(u)0}}{8L^{4/9}})[2c_s(\chi_s a_s) - c_d(\chi_d a_d)]a_u + (M^2 - \frac{m^2_{(s)0}}{8L^{4/9}})[2c_u(\chi_u a_u) + 2c_d(\chi_d a_d)]a_s\}+$$
$$\frac{L^{4/9}}{108}\{c_u(2\kappa_u - \xi_u)a_u(2a_s - a_d) + c_d(2\kappa_d - \xi_d)a_d(2a_s - a_u) + 2c_s(2\kappa_s - \xi_s)a_s(a_u + a_d)\},$$

which with $a_u = a_d = a$, $\chi_u = \chi_d = \chi$, $\kappa_u = \kappa_d = \kappa$, $\xi_u = \xi_d = \xi$, and $a_s/a = f + 1$, $(\chi_s a_s)/(\chi a) = \phi$, similar for ξ, κ, goes to

$$\to [-\frac{\chi a^2}{18L^{4/27}}(M^2 - \frac{m^2_{(s)0}}{8L^{4/9}}) + \frac{L^{4/9}}{108}(2\kappa - \xi)a^2][(c_u + c_d)(1 + 2f) + 4c_s\phi]$$

in accord with [7]. The difference $\hat{\Sigma}^{0(6)}_{su} - \hat{\Sigma}^{0(6)}_{sd}$ yields:

$$\sqrt{3}(\Sigma^0\Lambda)^{(6)} \to -(c_u - c_d)[\frac{-\chi}{6L^{4/27}}(M^2 - \frac{m^2_{(s)0}}{8L^{4/9}}) + \frac{L^{4/9}}{36}(2\kappa - \xi)]aa_s$$

which agrees with the result of [3].

Now we perform calculations of the 7th term given by the diagrams of the Fig.7:

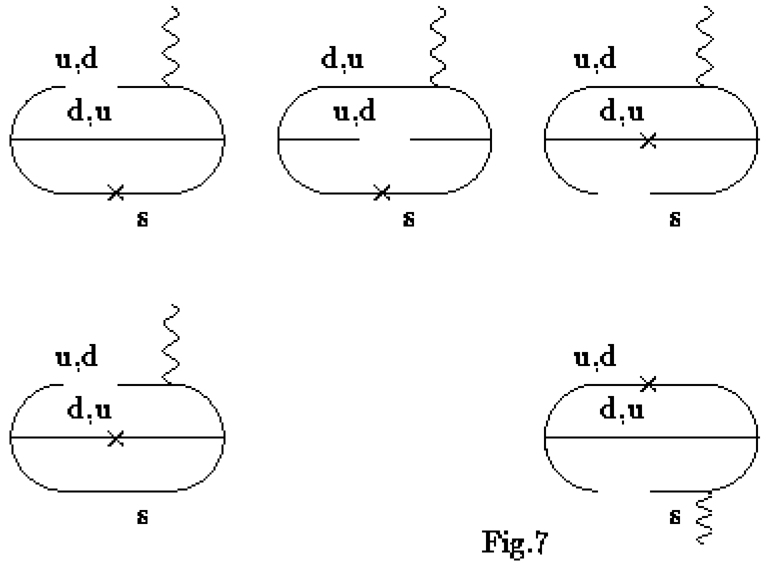

Fig.7

$$2\Sigma^{0(7)} = -\frac{M^2}{8L^{4/9}}4[2(c_u a_u + c_d a_d)m_s - (c_u a_d + c_d a_u)m_s +$$
$$2c_s(m_u + m_d)a_s + 2(c_u a_u m_d + c_d a_d m_u) - (c_u m_d + c_d m_u)a_s]. \quad (17)$$

Performing changes $s \leftrightarrow d$ and $s \leftrightarrow u$ to obtain the auxiliary quantities we get

$$2\hat{\Sigma}^{0(7)}_{sd} = -\frac{M^2}{8L^{4/9}}4[2(c_u a_u + c_s a_s)m_d - (c_u a_s + c_s a_u)m_d +$$
$$2c_d(m_u + m_s)a_d + 2(c_u a_u m_s + c_s a_s m_u) - (c_u m_s + c_s m_u)a_d],$$
$$2\hat{\Sigma}^{0(7)}_{su} = -\frac{M^2}{8L^{4/9}}4[2(c_d a_d + c_s a_s)m_u - (c_d a_s + c_s a_d)m_u +$$
$$2c_u(m_d + m_s)a_u + 2(c_d a_d m_s + c_s a_s m_d) - (c_d m_s + c_s m_d)a_u]$$

and using Eq.(11) we obtain

$$\Lambda^{(7)} = -\frac{M^2}{8L^{4/9}}\frac{2}{3}[6(c_u a_u m_d + c_d a_d m_u) + 6c_s a_s(m_u + m_d) - a_s(c_u m_d + c_d m_u) +$$
$$6m_s(c_u a_u + c_d a_d) - m_s(c_u a_d + c_d a_u)]$$
$$\to (\sim m_s, a_u = a_d = a) - \frac{15M^2}{36L^{4/9}}m_s(c_u + c_d)a,$$

and here it is the only discrepancy between our result and the corresponding term in [7], as they have (19/36) instead of our (15/36).The only divergence in the 7th term seems to be due either to some misprint or eventually to some diagram we have not taken into account in the Fig.7. But the example of all the other contributions including those given by the diagrams of the Fig.9 to see later (the contributions of which vanishes for all baryons but Λ in the limit of zero masses of the light quarks) teaches us that an eventually missed diagram would follow Eq.(11) all the same.

We have checked our result upon applying the Eq.(12) to the obtained expression for $\Lambda^{(7)}$ in order to arrive at the expression for $\Sigma^{0(7)}$. The answer have coincided with that of the Eq.(17).

The difference $\hat{\Sigma}^{0(7)}_{su} - \hat{\Sigma}^{0(7)}_{sd}$ ($\sim m_s$) upon using Eq.(13) yields:

$$\sqrt{3}(\Sigma^0\Lambda)^{(7)} = -\frac{M^2}{4L^{4/9}}[(c_u a_d - c_d a_u)m_s]$$

$$\to (\quad a_u = a_d = a)\frac{M^2}{L^{4/9}}(-1/4)(c_u - c_d)m_s a.$$

coefficient (-1/4) to compare with (1/3) in [3]. Here again we see a discrepancy between our result and that of [3], while as we have seen and shall see all other terms perfectly satisfy Eq.(13).

The 8th term comes from the diagrams of the Fig.8:

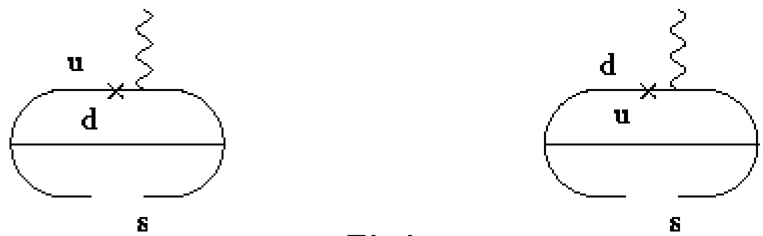

Fig.8

$$2\Sigma^{0(8)} = \frac{M^2}{2L^{4/9}}[ln[\frac{M^2}{\Lambda^2}] - 1 - \gamma_{EM}]2(c_u m_u + c_d m_d)a_s$$

wherefrom upon using Eq.(11):

$$\Lambda^{(8)} = \frac{M^2}{6L^{4/9}}[ln(\frac{M^2}{\Lambda^2}) - 1 - \gamma_{EM}][c_u m_u(2a_d - a_s) +$$

$$c_d m_d(2a_u - a_s) + 2c_s m_s(a_u + a_d)] \to \frac{2m_s a c_s M^2}{3L^{4/9}}[ln(\frac{M^2}{\Lambda^2}) - 1 - \gamma_{EM}]$$

in agreement with the result of [7, 9].

The difference $\hat{\Sigma}^{0(8)}_{su} - \hat{\Sigma}^{0(8)}_{sd}$ upon using Eq.(13) yields:

$$\sqrt{3}(\Sigma^0\Lambda)^{(8)} = \frac{M^2}{2L^{4/9}}[ln[\frac{M^2}{\Lambda^2}] - 1 - \gamma_{EM}][(c_u m_u a_d - c_d m_d a_u) + c_s m_s(a_d - a_u)] \Rightarrow 0.$$

The 9th term $\Sigma^{0(9)}$ comes from the diagrams of the Fig.9:

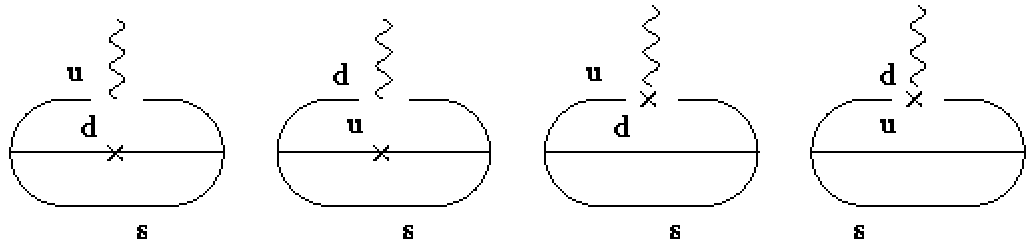

Fig.9

$$2\Sigma^{0(9)} = -\frac{M^4}{4L^{28/27}}[2(c_u a_u \chi_u m_d + c_d a_d \chi_d m_u) - 2(c_u a_u \chi_u m_u + c_d a_d \chi_d m_d)].$$

This term is identically zero with $m_u = m_d$. So a contribution to the $\Lambda^{(9)}$ term comes only from the auxiliary quantities $\tilde{\Sigma}^{0(9)}_{sd}$ and $\tilde{\Sigma}^{0(9)}_{su}$:

$$2\tilde{\Sigma}^{0(9)}_{sd} = -\frac{M^4}{4L^{28/27}}[2(c_u a_u \chi_u m_s + c_s a_s \chi_s m_u) - 2(c_u a_u \chi_u m_u + c_s a_s \chi_s m_s)],$$

$$2\tilde{\Sigma}^{0(9)}_{su} = -\frac{M^4}{4L^{28/27}}[2(c_d a_d \chi_d m_s + c_s a_s \chi_s m_d) - 2(c_d a_d \chi_d m_d + c_s a_s \chi_s m_s)].$$

As a result we get from Eq.(11):

$$\Lambda^{(9)} = -\frac{M^4}{12L^{28/27}}\{2[c_u a_u \chi_u + c_d a_d \chi_d - 2c_s a_s \chi_s]m_s - (c_u a_u \chi_u m_d + c_d a_d \chi_d m_u) -$$

$$(c_u a_u \chi_u m_u + c_d a_d \chi_d m_d) + 2c_s a_s \chi_s (m_u + m_d)\}$$

and for $m_u = m_d = 0, a_u = a_d = a, \chi_u = \chi_d = \chi, \chi_s a_s/\chi a = \phi$

$$\to -\frac{M^4}{6L^{28/27}}(c_u + c_d - 2c_s\phi)m_s a\chi$$

in accord with the Eq.(4) in [8].

The difference $\tilde{\Sigma}^{0(9)}_{su} - \tilde{\Sigma}^{0(9)}_{sd}$ upon using Eq.(13) yields:

$$\sqrt{3}(\Sigma^0\Lambda)^{(9)} = \frac{M^4}{4L^{28/27}}[m_s a\chi(c_u - c_d) + c_s a_s \chi_s (m_u - m_d) - (c_u m_u - c_d m_d)a\chi]$$

and for $m_u = m_d = 0$,

$$\to -\frac{M^2}{4L^{28/27}}m_s a\chi(c_u - c_d),$$

which agrees with the corresponding term in [3].

The 10th term $\Sigma^{0(10)}$ comes from the diagrams of the Fig.10:

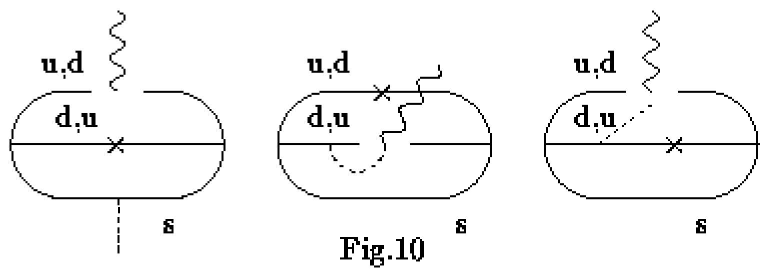

Fig.10

The last diagram contributes only to the κ term and has an infrared divergence regularized by a cutoff Λ. Finally,

$$2\Sigma^{0(10)} = [\frac{1}{6}(2\kappa_u - \xi_u))M^2 - \frac{M^2}{2}\kappa_u[ln[\frac{M^2}{\Lambda^2}] - 1 - \gamma_{EM}]]c_u a_u m_d +$$

$$[\frac{1}{6}(2\kappa_d - \xi_d))M^2 - \frac{M^2}{2}\kappa_d[ln[\frac{M^2}{\Lambda^2}] - 1 - \gamma_{EM}]]c_d a_d m_u.$$

Performing changes $(d \leftrightarrow s)$ and $(u \leftrightarrow s)$ to construct quantities $\tilde{\Sigma}^{0(10)}_{sd}$ and $\tilde{\Sigma}^{0(10)}_{su}$ and putting them into Eq.(11) we obtain for the contribution $\Lambda^{(10)}$

$$\Lambda^{(10)} = M^2[\frac{1}{36}(2\kappa_u - \xi_u)) - \frac{1}{12}\kappa_u[ln(\frac{M^2}{\Lambda^2}) - 1 - \gamma_{EM}]]c_u a_u(2m_s - m_d) +$$

$$M^2[\frac{1}{36}(2\kappa_d-\xi_d))-\frac{1}{12}\kappa_d[ln(\frac{M^2}{\Lambda^2})-1-\gamma_{EM}]]e_d a_d(2m_s-m_u)+$$
$$2M^2[\frac{1}{36}(2\kappa_s-\xi_s))-\frac{1}{12}\kappa_s[ln(\frac{M^2}{\Lambda^2})-1-\gamma_{EM}]]e_s a_s(m_u+m_d).$$

or (maintaining only terms $\sim m_s$ and putting $a_u=a_d=a$, $\kappa_u=\kappa_d=\kappa$, $\xi_u=\xi_d=\xi$):

$$\Lambda^{(10)}=(e_u+e_d)[\frac{1}{18}(2\kappa-\xi))M^2-\frac{M^2}{6}\kappa[ln(\frac{M^2}{\Lambda^2})-1-\gamma_{EM}]]am_s$$

again in full agreement with the results of [7],[9].

The corresponding term for $\Sigma^0\Lambda$ transition magnetic moment can be readily obtained through Eq.(13):

$$\sqrt{3}(\Sigma^0\Lambda)^{(10)}=[\frac{1}{12}(2\kappa-\xi)M^2-\frac{M^2}{4}\kappa[ln(\frac{M^2}{\Lambda^2})-1-\gamma_{EM}]][m_s a(e_u-e_d)+$$

$$e_s a_s\chi_s(m_u-m_d)]\rightarrow(e_u-e_d)[\frac{1}{12}(2\kappa-\xi)M^2-\frac{M^2}{4}\kappa[ln(\frac{M^2}{\Lambda^2})-1-\gamma_{EM}]]m_s a.$$

Combining all the terms $\Lambda^{(k)}$, $k=1,...,10$, we arrive at the QCD Borel sum rule for the Λ magnetic moment:

$$\frac{M^6}{12L^{4/9}}(e_u+e_d+4e_s)+\frac{M^2b}{144L^{4/9}}(4e_u+4e_d+7e_s)$$
$$-\frac{L^{4/9}}{108M^2}[2(7e_u+7e_d+e_s)+8f(2e_u+2e_d+e_s)]a^2-$$
$$\frac{M^2b}{192L^{4/9}}[ln(\frac{M^2}{\Lambda^2})-1-\gamma_{EM}](5e_u+5e_d+2e_s)-$$
$$\frac{M^2b}{108L^{4/9}}[ln(\frac{M^2}{\Lambda^2})-\gamma_{EM}-\frac{M^2}{2\Lambda^2}](e_u+e_d+4e_s)+$$
$$[\frac{-\chi a^2}{18L^{4/27}}(M^2-\frac{m^2_{(s)0}}{8L^{4/9}})+\frac{L^{4/9}}{108}(2\kappa-\xi)a^2][(e_u+e_d)(1+2f)+4e_s\phi]$$
$$-\frac{15M^2}{36L^{4/9}}m_s(e_u+e_d)a+\frac{2m_s a e_s M^2}{3L^{4/9}}[ln(\frac{M^2}{\Lambda^2})-1-\gamma_{EM}]$$
$$-\frac{M^2}{6L^{28/27}}(e_u+e_d-2e_s\phi)m_s a\chi+$$
$$(e_u+e_d)[\frac{1}{18}(2\kappa-\xi))M^2-\frac{M^2}{6}\kappa[ln(\frac{M^2}{\Lambda^2})-1-\gamma_{EM}]]am_s$$
$$=\beta^2_\Lambda\mu_\Lambda e^{-M^2_\Lambda/M^2}(1+A_\Lambda M^2)+e.s.c.$$

in agreement with [7] but the coefficient (15/36) instead of (19/36) in the 7th term.

Combining all the terms $(\Sigma^0\Lambda)^k, k = 1, ...10$, we arrive at the QCD Borel sum rule for the $\Sigma^0\Lambda$ transition magnetic moment:

$$
\begin{aligned}
&(e_u - e_d)[\frac{M^6}{4L^{4/9}} + \frac{L^{4/9}}{9M^2}a_s a + \frac{M^2 b}{24L^{4/9}} \\
&-\frac{M^2 b}{144L^{4/9}}[ln(\frac{M^2}{\Lambda^2}) - 1 - \gamma_{EM}] + 4[ln(\frac{M^2}{\Lambda^2}) - \gamma_{EM} - \frac{M^2}{2\Lambda^2}] + [\\
&\frac{-\chi}{6L^{4/27}}(M^2 - \frac{m^2_{(s)0}}{8L^{4/9}}) + \frac{L^{4/9}}{36}(2\kappa - \xi)]a a_s + \\
&\frac{M^2}{L^{4/9}}\frac{1}{4}m_s a - \frac{M^4}{4L^{28/27}}m_s a\chi + [\frac{1}{18}(2\kappa - \xi)M^2 - \\
&\frac{M^2}{6}\kappa[ln(\frac{M^2}{\Lambda^2}) - 1 - \gamma_{EM}]]m_s a = \beta_\Sigma\beta_\Lambda\sqrt{3}\mu_{\Sigma^0\Lambda}e^{-m^2/M^2}(1 + A_{\Sigma^0\Lambda}M^2) + c.s.c. \qquad (18)
\end{aligned}
$$

5 Conclusion

We have shown on the example of QDC Borel sum rules for magnetic moments of the Σ and Λ hyperons in the version proposed in [7] that starting from the sum rule for the Σ hyperon it is straightforward to obtain the corresponding sum rule for the Λ hyperon upon using intercrossed relation Eq.(11) as well as to construct a corresponding sum rule for the $\Lambda - \Sigma^0$ transition magnetic moment.

We have also checked without writing it down that starting from the obtained QCD sum rule for the Λ magnetic moment and applying Eq.(12) we return to the initial sum rule for the Σ^0 magnetic moment.

We have checked these formulae for the QCD Light-Cone sum rules written by one of us with coauthors [5] and seen that the Eqs.((11,12) are satisfied exactly. The results will be published elsewhere. But we think that it is more convincing and instructive to take already published results in order to prove our formulae. Earlier similar result has been proved for the QCD mass sum rules [10] upon relating Σ^0 and Λ mass sum rules where the agreement with the sum rules of [1] has been found to be perfect.

As for magnetic moment sum rules we have shown explicitly that every group of diagrams for Σ^0 generates according to Eqs.(11,13) analogous groups of diagrams for Λ and $\Sigma^0\Lambda$ in practically full agreement with the results of [8] and [3]. (The only our divergence with [8] in the 7th term seems to be due either to some misprint or eventually to some diagram we have not taken into account.)

The proposed relations Eqs.(11-14) can be used not only to obtain many properties of the the Σ-like baryons from those of Λ-like ones *et vice versa* but also to check mutually many-terms relations for the Σ-like and Λ-like baryons.

Acknowledgments

We thank B.L.Ioffe for fruitful discussion. We are grateful to T.Aliev, F.Hussain, G.Thompson for their interest to the work.

One of us (V.Z.) is grateful to Prof.S.Randjbar-Daemi for the hospitality extended to him at HE section of ICTP (Trieste, Italy). The financial support of the ICTP in the initial stage of the work is greatefully acknowledged.

References

[1] W-Y.P.Hwang and K.-C.Yang, Phys.Rev.D**49**,460 (1994).

[2] K.-C.Yang, W-Y.Hwang, E.M.Henley, and L.S.Kisslinger, Phys.Rev.D**47**,3001 (1993).

[3] Shi-lin Zhu, W-Y.P.Hwang and Ze-sen Yang, Phys.Rev.D**57**, 1527 (1998).

[4] T.M.Aliev, A.Özpineci, M.Savci, Phys.Rev.D**62**, 053012 (2000).

[5] T.M.Aliev, A.Özpineci, M.Savci, Nucl. Phys. **A678**, 443 (2000).

[6] V.M.Belyaev, B.L.Ioffe, JETP **56**, 493 (1982); B.L.Ioffe, Smilga, Nucl.Phys. **B232**, 109 (1984); I. I. Balitsky and A. V. Yung, Phys. Lett. **B129**, 328 (1983).

[7] Ch.B.Chiu, J.Pasupathy, S.L.Wilson, Phys.Rev.D**33**,1961 (1986).

[8] Ch.B.Chiu, J.Pasupathy, S.L.Wilson, Phys.Rev.D**36**,1442 (1987).

[9] Ch.B.Chiu, S.L.Wilson, J.Pasupathy, J.P.Singh, Phys.Rev.D**36**,1553 (1987).

[10] A.Özpineci, S.B.Yakovlev, V.S.Zamiralov, NPI MSU Preprint 2003-18/731, Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia.

Алтуг Озшинечи
Станислав Борисович Яковлев
Валерий Семенович Замиралов

Правила сумм КХД: Перекрестные соотношения для магнитных моментов Σ^0 и Λ

Работа поступила в ОНТИ 1.11.2003

ИД 00545 от 06.12.1999

Издательство
Учебно-научного центра довузовского образования

117246, Москва, ул. Обручева,55А
Тел./факс(095)718-6966
e-mail:izdat@abiturcenter.ru
http:/www.abiturcenter.ru

Гигиенический сертификат 77.99.2.925.П.9139.2.00 от 24.02.2000
Налоговые льготы - Общероссийский классификатор продукции
ОК-005-93, том1 - 953000

Заказное. Подписано к печати 1.11.2003 г. Формат 60×90/16
Бумага офсетная 2. Усл.печ.л. 1,1
Тираж 50 экз. Заказ 345
Отпечатано в Мини-типографии УНЦ ДО
в полном соответствии с качеством
предоставленного оригинал-макета