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 $\Sigma^0 - \Lambda$  Mass Splitting

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Splitting

**Summary**

New relations between QCD Borel sum rules for masses of  $\Sigma^0$  and  $\Lambda$  hyperons are constructed. It is shown that starting from the sum rule for the  $\Sigma^0$  hyperon mass it is straightforward to obtain the corresponding sum rule for the  $\Lambda$  hyperon mass *et vice versa*.

$\Sigma^0 - \Lambda$  , ,  $\Sigma^0$  ,  $\Lambda$  *et vice versa*

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## 1.Introduction

Recently a series of papers were dedicated to study hadron properties of the  $\Sigma, \Sigma_c$  baryons as well as of the  $\Lambda, \Lambda_c$  ones in the framework of various QCD sum rules [1], [2], [3], [4], [5] which have their origin in the works [6], [7]. Many interesting results were obtained. But full expressions for mass or magnetic moment sum rules often become too long and tedious to achieve and prove. Is it possible to relate all these results among themselves and derive, say,  $\Lambda$  hyperon properties from that of  $\Sigma$  ones *et vice versa* or just to check them mutually?

We propose here nonlinear intercrossed relations which relate matrix elements of  $\Sigma$ -like baryons with those of  $\Lambda$ -like ones *et vice versa*. Their origin lies in the relation between isotopic,  $U$ - and  $V$ -spin quantities and is quasi obvious in the framework of the quark model. These relations prove to be valid for any QCD sum rules and seem to be useful while obtaining hadron properties of the  $\Lambda$ -like baryons from those of the  $\Sigma$ -like baryons ( and *vice versa*) or checking expressions for them reciprocally. The latter proves to be important as final QCD SR's comes to be rather long and cumbersome so it becomes a difficult work to compare or prove them term by term.

## 2.Relation between magnetic moments of hyperons $\Sigma^0$ and $\Lambda$ in the NRQM

We begin with a simple example. Let us write magnetic moments of hyperons  $\Sigma^0$  and  $\Lambda$  of the baryon octet in the NRQM:

$$\mu(\Sigma^0(ud, s)) = \frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_s; \quad \mu(\Lambda) = \mu_s. \quad (1)$$

As it is known magnetic moment of any other baryon of the octet but that of the  $\Lambda$  hyperon can be obtained from the expression for the  $\Sigma^0$ .

E.g., magnetic moment of the  $\Sigma^+(uu, s)$  hyperon is obtained just by putting  $\mu_u$  instead of  $\mu_d$  in Eq.(1):

$$\mu(\Sigma^+) = \frac{4}{9}\mu_u - \frac{1}{9}\mu_s.$$

But magnetic moment of the  $\Lambda$  hyperon can be also obtained from that of the  $\Sigma^0$  one, as well as magnetic moment of the  $\Sigma^0$  can be obtained from that of the  $\Lambda$  one. For that purpose let us formally perform in Eq.(1) the exchange  $d \leftrightarrow s$  to get

$$\mu(\tilde{\Sigma}_{d \leftrightarrow s}^0) = \frac{2}{9}\mu_u + \frac{2}{9}\mu_s - \frac{1}{9}\mu_d; \quad \mu(\tilde{\Lambda}_{u \leftrightarrow s}) = \mu_d \quad (2)$$

and the exchange  $u \leftrightarrow s$  to get

$$\mu(\tilde{\Sigma}_{u \leftrightarrow s}^0) = \frac{2}{9}\mu_d + \frac{2}{9}\mu_s - \frac{1}{9}\mu_u; \quad \mu(\tilde{\Lambda}_{u \leftrightarrow s}) = \mu_u. \quad (3)$$

The following relations are valid:

$$\begin{aligned} 2(\mu(\tilde{\Sigma}_{d \leftrightarrow s}^0) + \mu(\tilde{\Sigma}_{u \leftrightarrow s}^0)) - \mu(\Sigma^0) &= 3\mu(\Lambda); \\ 2(\mu(\tilde{\Lambda}_{d \leftrightarrow s}) + \mu(\tilde{\Lambda}_{u \leftrightarrow s})) - \mu(\Lambda) &= 3\mu(\Sigma^0). \end{aligned} \quad (4)$$

The origin of these relations lies in the structure of baryon wave functions in the NRQM with isospin  $I = 1, 0$  and  $I_3 = 0$ :

$$2\sqrt{3}|\Sigma^0(ud, s)\rangle_{\uparrow=}$$

$$|2u_{\uparrow}d_{\uparrow}s_{\downarrow} + 2d_{\uparrow}u_{\uparrow}s_{\downarrow} - u_{\uparrow}s_{\uparrow}d_{\downarrow} - s_{\uparrow}u_{\uparrow}d_{\downarrow} - d_{\uparrow}s_{\uparrow}u_{\downarrow} - s_{\uparrow}d_{\uparrow}u_{\downarrow}\rangle,$$

$$2|\Lambda\rangle_{\uparrow=} |d_{\uparrow}s_{\uparrow}u_{\downarrow} + s_{\uparrow}d_{\uparrow}u_{\downarrow} - u_{\uparrow}s_{\uparrow}d_{\downarrow} - s_{\uparrow}u_{\uparrow}d_{\downarrow}\rangle,$$

where  $q_{\uparrow}$  ( $q_{\downarrow}$ ) means wave function of the quark  $q$  (here  $q = u, d, s$ ) with the helicity  $+1/2$  ( $-1/2$ ). With the exchanges  $d \leftrightarrow s$  and  $u \leftrightarrow s$  one arrives at the corresponding  $U$ -spin and  $V$ -spin quantities, so  $U = 1, 0$  and  $U_3 = 0$  baryon wave functions are

$$-2|\tilde{\Sigma}_{d \leftrightarrow s}^0(us, d)\rangle = |\Sigma^0(ud, s)\rangle + \sqrt{3}|\Lambda\rangle,$$

$$-2|\bar{\Lambda}_{d\leftrightarrow s}\rangle = -\sqrt{3}|\Sigma^0(ud, s)\rangle + |\Lambda\rangle,$$

while  $V = 1, V_3 = 0$  and  $V = 0$  baryon wave functions are

$$-2\bar{\Sigma}_{u\leftrightarrow s}^0(ds, u) = |\Sigma^0(ud, s)\rangle - \sqrt{3}|\Lambda\rangle,$$

$$2|\bar{\Lambda}_{u\leftrightarrow s}\rangle = \sqrt{3}|\Sigma^0(ud, s)\rangle + |\Lambda\rangle.$$

It is easy to show that relations given by Eqs.(2,3) immediately follow.

### 3.Relation between QCD correlators for $\Sigma^0$ and $\Lambda$ hyperons

Now we demonstrate how similar considerations work for QCD sum rules on the example of QCD Borel mass sum rules.

The starting point would be two-point Green's function for hyperons  $\Sigma^0$  and  $\Lambda$  of the baryon octet:

$$\Pi^{\Sigma^0, \Lambda} = i \int d^4x e^{ixz} \langle 0 | T \{ \eta^{\Sigma^0, \Lambda}(x), \bar{\eta}^{\Sigma^0, \Lambda}(0) \} | 0 \rangle, \quad (5)$$

where isovector (with  $I_3 = 0$ ) and isoscalar field operators could be chosen as [3]

$$\begin{aligned} \eta^{\Sigma^0} &= \frac{1}{2} \epsilon_{abc} [(u^{aT} C s^b) \gamma_5 d^c - (d^{aT} C s^b) \gamma_5 u^c - \\ &\quad (u^{aT} C \gamma_5 s^b) d^c + (d^{aT} C \gamma_5 s^b) u^c], \\ \eta^\Lambda &= \frac{1}{2\sqrt{3}} \epsilon_{abc} [-2(u^{aT} C d^b) \gamma_5 s^c + (u^{aT} C s^b) \gamma_5 d^c \\ &\quad + (d^{aT} C s^b) \gamma_5 u^c + 2(u^{aT} C \gamma_5 d^b) s^c \\ &\quad - (u^{aT} C \gamma_5 s^b) d^c - (d^{aT} C \gamma_5 s^b) u^c], \end{aligned} \quad (6)$$

where  $a, b, c$  are the color indices and  $u, d, s$  are quark wave functions,  $C$  is charge conjugation matrix,

We show now that one can operate with  $\Sigma$  hyperon and obtain the results for the  $\Lambda$  hyperon. The reasoning would be valid also for charm and beauty  $\Sigma$ -like and  $\Lambda$ -like baryons.

In order to arrive at the desired relations we write not only isospin quantities but also  $U$ -spin and  $V$ -spin ones.

Let us introduce  $U$ -vector (with  $U_3 = 0$ ) and  $U$ -scalar field operators just formally changing ( $d \leftrightarrow s$ ) in the Eq.(7):

$$\begin{aligned}\tilde{\eta}^{\Sigma^0(d\leftrightarrow s)} &= \frac{1}{2}\epsilon_{abc}[(u^{aT}C d^b)\gamma_5 s^c - (s^{aT}C d^b)\gamma_5 u^c \\ &\quad - (u^{aT}C\gamma_5 d^b)s^c + (s^{aT}C\gamma_5 d^b)u^c], \\ \tilde{\eta}^{\Lambda(d\leftrightarrow s)} &= \frac{1}{2\sqrt{3}}\epsilon_{abc}[-2(u^{aT}C s^b)\gamma_5 d^c + \\ &\quad (u^{aT}C d^b)\gamma_5 s^c + (s^{aT}C d^b)\gamma_5 u^c + 2(u^{aT}C\gamma_5 s^b)d^c \\ &\quad - (u^{aT}C\gamma_5 s^b)d^c - (s^{aT}C\gamma_5 d^b)u^c],\end{aligned}\quad (7)$$

Similarly we introduce  $V$ -vector (with  $V_3 = 0$ ) and  $V$ -scalar field operators just changing ( $u \leftrightarrow s$ ) in the Eq.(7):

$$\begin{aligned}\tilde{\eta}^{\Sigma^0(u\leftrightarrow s)} &= \frac{1}{2}\epsilon_{abc}[(s^{aT}C u^b)\gamma_5 d^c - (d^{aT}C u^b)\gamma_5 s^c \\ &\quad - (s^{aT}C\gamma_5 u^b)d^c + (d^{aT}C\gamma_5 u^b)s^c], \\ \tilde{\eta}^{\Lambda(u\leftrightarrow s)} &= \frac{1}{2\sqrt{3}}\epsilon_{abc}[-2(s^{aT}C d^b)\gamma_5 u^c + \\ &\quad (s^{aT}C u^b)\gamma_5 d^c + (d^{aT}C u^b)\gamma_5 s^c + 2(s^{aT}C\gamma_5 d^b)u^c \\ &\quad - (s^{aT}C\gamma_5 d^b)u^c - (d^{aT}C\gamma_5 u^b)s^c],\end{aligned}\quad (8)$$

Field operators of the Eq.(7) and Eq.(8) can be related through

$$\begin{aligned}-2\tilde{\eta}^{\Lambda(d\leftrightarrow s)} &= \eta^{\Lambda} + \sqrt{3}\eta^{\Sigma^0}, \\ 2\tilde{\eta}^{\Sigma^0(d\leftrightarrow s)} &= \sqrt{3}\eta^{\Lambda} - \eta^{\Sigma^0}, \\ 2\tilde{\eta}^{\Lambda(u\leftrightarrow s)} &= \eta^{\Lambda} - \sqrt{3}\eta^{\Sigma^0}, \\ -2\tilde{\eta}^{\Sigma^0(u\leftrightarrow s)} &= \sqrt{3}\eta^{\Lambda} + \eta^{\Sigma^0},\end{aligned}\quad (9)$$

Upon using Eqs.(7,7,10) two-point Green functions of the Eq.(5) for hyperons  $\Sigma^0$  and  $\Lambda$  of the baryon octet can be related as

$$2[\tilde{\Pi}^{\Sigma^0(d\leftrightarrow s)} + \tilde{\Pi}^{\Sigma^0(u\leftrightarrow s)}] - \Pi^{\Sigma^0} = 3\Pi^\Lambda, \quad (10)$$

$$2[\tilde{\Pi}^\Lambda(d\leftrightarrow s) + \tilde{\Pi}^\Lambda(u\leftrightarrow s)] - \Pi^\Lambda = 3\Pi^{\Sigma^0}. \quad (11)$$

These are essentially nonlinear relations.

It is seen that starting calculations, e.g., from  $\Sigma$ -like quantities one arrives at the corresponding quantities for  $\Lambda$ -like baryons and *vice versa*.

#### 4. Intercrossed relations for the QCD Borel mass sum rules

In order to see how it works we have repeated calculations of the first of the QCD mass sum rules for the  $\Sigma^0$  hyperon following [1] conserving non-degenerated quantities for  $u$  and  $d$  quarks, namely

$$\begin{aligned} & \frac{M^6}{8} L^{-4/9} E_2 + \frac{bM^2}{32} L^{-4/9} E_0 + \frac{a_u a_d}{6} L^{4/9} - \\ & \frac{a_u a_d (m_{0(u)}^2 + m_{0(d)}^2)}{48M^2 L^{2/27}} - \frac{m_s a_s m_{0(s)}^2}{24L^{26/27}} - \\ & \frac{M^2 E_0}{4L^{4/9}} [a_s m_s - (a_u - a_d)(m_d - m_u)] - \\ & \frac{1}{48} [3m_u a_d m_{0(d)}^2 + 3m_d a_u m_{0(u)}^2 - m_u a_u m_{0(u)}^2 - \\ & - m_d a_d m_{0(d)}^2] L^{-26/27} = \beta_{\Sigma^0}^2 e^{-(M_{\Sigma^0}^2/M^2)} + e.s.c., \end{aligned} \quad (12)$$

where [5]

$$\begin{aligned} a_q &= -(2\pi)^2 \langle \bar{q}q \rangle, \quad b = \langle g_c G^2 \rangle, \\ a_q m_{0(q)}^2 &= (2\pi)^2 \langle g_c \bar{q}\sigma \cdot Gq \rangle, \quad q = u, d, s. \\ L &= \ln(M^2/\Lambda_{QCD}^2)/\ln(\mu^2/\Lambda_{QCD}^2), \end{aligned} \quad (13)$$



$$E_n(x) = 1 - e^{-x}(1 + x + \dots + x^n/n!),$$

$$x = W_B^2/M^2, \quad B = \Sigma^0, \Lambda.$$

Borel residue for the  $\Lambda$  hyperon is defined as

$$\langle 0 | \eta^\Lambda(0) | \Lambda(p) \rangle = \lambda_\Lambda u(p), \quad 4\beta_\Lambda^2 = (2\pi)^4 \lambda_\Lambda^2,$$

and similarly for the  $\Sigma$  hyperon, while e.s.c. means 'excited-state contributions'.

The renormalization scale  $\mu$  is taken usually to be around 500 MeV while QCD scale parameter should be around 100 MeV. With  $m_{0(u)} = m_{0(d)}$  in Eq.(12) one returns to the expression given by Eq.(21) in [1].

Now changing ( $d \leftrightarrow s$ ) and ( $u \leftrightarrow s$ ) in the LHS ( $\Sigma^0$ ) of the Eq.(12) to obtain LHS ( $\bar{\Sigma}^0(d \leftrightarrow s)$ ) and LHS ( $\bar{\Sigma}^0(u \leftrightarrow s)$ ), respectively, and using Eq.(10) we obtain for the  $\Lambda$ -mass SR:

$$\begin{aligned} & \frac{M^6}{8L^{4/9}} E_2 + \frac{bM^2}{32L^{4/9}} E_0 + \frac{2a_s(a_u + a_d) - a_u a_d}{18} L^{4/9} - \\ & \frac{L^{-2/27}}{144M^2} [2(a_u + a_d)a_s m_{0(s)}^2 + 2(m_{0(u)}^2 a_u + m_{0(d)}^2 a_d) - \\ & a_u a_d (m_{0(u)}^2 + m_{0(d)}^2)] - \frac{M^2 E_0 m_s}{12L^{4/9}} [3a_s - 2(a_u + a_d)] \\ & - \frac{M^2}{12L^{4/9}} E_0 [3(m_u a_u + m_d a_d) + m_d a_u + m_u a_d - \\ & 2(m_u + m_d)a_s] - \frac{1}{48} [2m_s(a_u m_{0(u)}^2 + a_d m_{0(d)}^2) - \\ & (a_u m_{0(u)}^2 - a_d m_{0(d)}^2)(a_u - a_d)] L^{-26/27} - \quad (14) \\ & \frac{1}{24} (m_u + m_d - m_s) a_s m_{0(s)}^2 L^{-26/27} = \beta_\Lambda^2 e^{-(M_\Lambda^2/M^2)} + e.s.c. \end{aligned}$$

With  $m_{0(u)} = m_{0(d)} = m_0$  in Eqs.(15) one returns to the expressions given by Eq.(23) in [1].

If also  $a_{0(u)} = a_{0(d)} = a$ ,  $m_0^2 = m_{0(s)}^2$  and  $m_u = m_d = 0$ , one returns to mass sum rules of [6] in the form given by Eq.(3) in [10] upon

neglecting factors  $L^{-2/27}$  and  $L^{-26/27}$  in two terms at the LHS:

$$\begin{aligned} & \frac{M^6}{8L^{4/9}} + \frac{bM^2}{32}L^{-4/9} + \frac{a^2}{6}L^{4/9} - \frac{a^2m_0^2}{24M^2L^{2/27}} - \\ & \frac{a_s m_s M^2}{4L^{4/9}} - \frac{m_s a_s m_0^2}{24L^{26/27}} = \beta_{\Sigma^0}^2 e^{-(M_{\Sigma^0}^2/M^2)} + e.s.c., \end{aligned} \quad (15)$$

$$\begin{aligned} & \frac{M^6}{8L^{4/9}} + \frac{bM^2}{32L^{4/9}} - \frac{a^2m_0^2}{24M^2L^{2/27}} + \\ & \frac{a_s m_s M^2}{12L^{4/9}} - \frac{m_s a_s m_0^2}{24L^{26/27}} = \beta_{\Lambda}^2 e^{-(M_{\Lambda}^2/M^2)} + e.s.c. \end{aligned} \quad (16)$$

Now we show the validity of the second intercrossed relation Eq.(11).

We start now from the QCD Borel mass sum rule for the  $\Lambda$  given by the Eq.(24) in [1]:

$$\begin{aligned} & \frac{M^4}{12}(2a_u + 2a_d - a_s)E_1 - \frac{b}{16}(2a_u + 2a_d - a_s) + \\ & \frac{\alpha_s}{\pi} \frac{L^{-1/9}}{243M^2} [108a_u a_d a_s + a_s(a_u^2 + a_d^2) \\ & \quad - 2(a_u a_d + a_s^2)(a_u + a_d)] \\ & \left( \frac{M^6 E_2}{12L^{8/9}} - \frac{bM^2 E_0}{96L^{8/9}} \right) (2m_u + 2m_d - m_s) + \\ & \frac{1}{36} [12m_s a_u a_d - 2m_s a_s (a_u + a_d)] + \\ & \frac{1}{36} [12a_s (m_u a_d + m_d a_u) + a_s (m_u a_u + m_d a_d) \\ & \quad - 2(m_u + m_d) a_u a_d] = \beta_{\Lambda}^2 M_{\Lambda} e^{-(M_{\Lambda}^2/M^2)} + e.s.c. \end{aligned} \quad (17)$$

Performing changes  $s \leftrightarrow d$  ( $s \leftrightarrow d$ ) we arrive at the corresponding Borel sum rules for  $\bar{\Lambda}_{d \leftrightarrow s}$  ( $\bar{\Lambda}_{u \leftrightarrow s}$ ).

Putting these expressions into Eq.(11) it is straightforward to obtain

$$\frac{a_s M^4}{4} E_1 - \frac{a_s b}{72} + \frac{\alpha_s}{\pi} \frac{L^{-1/9}}{81M^2} [-(a_u^2 + a_d^2) +$$

$$\begin{aligned}
& 36a_u a_d] a_s + \frac{M^6}{4L^8/9} m_s E_2 - \frac{bM^2}{32L^8/9} m_s E_0 + \\
& \frac{1}{12} a_s (4m_u a_d + 4m_d a_u - m_u a_u - m_d a_d) \\
& + \frac{1}{9} m_s a_u a_d = \beta_{\Sigma^0}^2 M_{\Sigma^0} e^{-(M_{\Sigma^0}^2/M^2)} e.s.c.,
\end{aligned} \tag{18}$$

which is just the relation given by the Eq.(22) in [1].

## 5. Conclusion

We have shown that starting from the QDC Borel mass sum rules for the  $\Sigma$  hyperon it is straightforward to obtain the corresponding quantities for the  $\Lambda$  hyperon and *vice versa* upon using intercrossed relations of the type given by Eqs.(2,3) and Eqs.(10,11).

More generally these relations can be used not only to obtain properties of the the  $\Sigma$ -like baryons from those of  $\Lambda$ -like ones and *vice versa* but also to check reciprocally many-terms relations for the  $\Sigma$ -like and  $\Lambda$ -like baryons.

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